MCV4U Homework

Critical Points, Local Maxima & Local Minima

Section A: Problem-solving Questions

1. Find the critical points for each function. Use the first derivative test to determine whether the critical point is a local maximum, local minimum, or neither.

(a). $f(x) = (x - 5)^{\frac{1}{3}}$

(b). $f(x) = \frac{2x}{x^2+9}$

2. Consider the function $f(x) = 3x^4 + ax^3 + bx^2 + cx + d$.

a. Find constants $a,\ b,\ c,$ and d such that the graph of f will have horizontal tangents at $(-2, -73,)$ and $(0, 9)$.

b. There is a third point that has a horizontal tangent. Find this point.

c. For all three points, determine whether each corresponds to a local maximum, a local minimum, or neither.

3. For a particular function $f'(x) = x^3 - 2x^2$.

a. For which values of x does $f'(x) = 0$?

b. Find the intervals of increase and decrease for $f(x)$.

4. Determine values of $a,\ b$ and c such that the graph of $y = ax^2 + bx + c$ has a relative maximum at $(3, 12)$ and crosses the y-axis at $(0, 1)$.

5. A section of roller coaster is in the shape of $f(x) =- x^3 - 2x^2 + x + 15$ where x is between -2 and $+2$.

a. Find all local extrema and explain what portions of the roller coaster they represent.

b. Is the highest point of this section of the ride at the beginning, the end, or neither?

Answer Key

1. a) $f'(x) = \frac{1}{3(x-5)^{\frac{2}{3}}}$

   When $f'(x) = undefined, x = 5$

   $f(5) = 0$

-1	$f'(x) = 0$	$+1$	**Nature**
$f'(4) =- 1$	5	$f'(6) = 1$	$- \ 0 \ +$

✅ $(5, 0)$ is a cusp.

b) $f'(x) = \frac{2(9-x^2)}{(x^2+9)^2}$

   When $f'(x) = 0, x = \pm 3$

   $f(3) =\pm \frac{1}{3}$

-1	$f'(x) = 0$	$+1$	**Nature**
$f'(2) = \frac{2}{5}$	3	$f'(4) =- 0.0224$	$+ \ 0 \ -$
$f'(-4) =- 0.0224$	-3	$f'(-2) = 0.059$	$- \ 0 \ +$

✅ $(3, \frac{1}{3})$ is maximum point and $(-3, -\frac{1}{3})$ is a minimum point.

2. $f'(x) = 12x^3 + 3ax^2 + 2bx + c$

   a) $a =- 4, \ b =- 36, \ c = 0$

   b) $(3, \ -198)$

   c) Local minimum: $(-2, -73)$ and $(3, -198)$
   Local maximum: $(0, -9)$

3. a) $x = 0$ and $x = 2$

   b) Decreasing intervals: $(-\infty, 2)$
   Increasing intervals: $(2, \infty)$

4. $a =- \frac{11}{9}, b = \frac{22}{3}, c = 1$

5. The coaster starts down a hill from $x =- 2$, reaching a local minimum at the bottom of a hill at $(- 1.55, 12.37)$. It then increases height until it reaches a local maximum at the top of a hill at $(0.22, 15.11)$. It then continues downward until $x = 2$.