

## Homework

1) Find vertical asymptotes of the following functions:

a)  $f(x) = \frac{x+1}{x+3}$

b)  $f(x) = \frac{2x}{(x-1)(x+4)}$

c)  $f(x) = \frac{3}{x^2-9}$

2) Find horizontal asymptotes of the following functions:

a)  $f(x) = \frac{2x+1}{3x-5}$

b)  $f(x) = \frac{x^2-4}{x^3+x}$

c)  $f(x) = \frac{x^3+1}{x+2}$

3) Find oblique asymptotes of the following functions:

a)  $f(x) = \frac{x^2+2x+1}{x}$

b)  $f(x) = \frac{2x^2-3x+5}{x-1}$

c)  $f(x) = \frac{x^3-1}{x^2}$

4) Find the asymptotes of the following functions:

d)  $f(x) = \frac{3x}{(x-2)(x+1)}$

e)  $f(x) = \frac{x^2-4}{x+1}$

5) Determine the behavior of the function  $f(x) = \frac{1}{(x-3)}$  near the vertical asymptote.

6) Determine the behavior of the function  $f(x) = \frac{3x}{x^2+1}$  near the horizontal asymptote.

7) Determine the behavior of the function  $f(x) = \frac{x^2+1}{x+1}$  near the oblique asymptote.

## Answer Key

1)

- a) V.A:  $x = -3$
- b) V.A:  $x = 1$  and  $x = -4$
- c) V.A:  $x = 3$  and  $x = -3$

2)

- a) H.A:  $y = \frac{2}{3}$  (Degree of numerator = Degree of denominator)
- b) H.A:  $y = 0$  (Degree of numerator < Degree of denominator)
- c) H.A:  $y = \text{DNE}$  (Degree of numerator > Degree of denominator)

3)

- a) O.A:  $y = x + 2$  (Long Division)
- b) O.A:  $y = 2x - 1$  (Long Division)
- c) O.A:  $y = x$  (Long Division)

4)

- a) V.A:  $x = 2$  and  $x = -1$ , H.A:  $y = 0$
- b) V.A:  $x = -1$ , O.A:  $y = x - 1$

5)

$$5) \quad f(x) = \frac{1}{(x-3)}$$

$$x-3 = 0 \\ x = 3 \text{ (V.A.)}$$

$$\lim_{x \rightarrow 3^-} f(x) = ? \quad \text{let } x = 2.9 \\ f(x) \approx (-)$$

$$\lim_{x \rightarrow 3^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 3^+} f(x) = ? \quad \text{let } x = 3.1 \\ f(x) = +$$

$$\lim_{x \rightarrow 3^+} f(x) = \infty$$

6)

$$f(x) = \frac{2x + 3}{x + 5}$$

$$\text{H.A: } \frac{2}{1} = 2$$

$$y = 2$$

$$\text{let } x = -1000$$

$$f(-1000) = 2.01$$

$$x \rightarrow -\infty$$

$$y \rightarrow 2 \text{ from above}$$

$$\text{let } x = 1000$$

$$f(1000) = 1.99$$

$$x \rightarrow \infty$$

$$y \rightarrow 2 \text{ from below}$$

7)

$$f(x) = \frac{x^2 + 1}{x + 1}$$

$$x+1 \overline{) \begin{array}{r} x^2 + 0x + 1 \\ \underline{x^2 + 1x} \phantom{0} \\ -x + 1 \\ \underline{-x + 1} \\ 2 \end{array}}$$

$$g(x) = x - 1$$

$$\text{let } x = 1000$$

$$f(1000) = 999.001 \quad g(x) = 999$$

$$x \rightarrow \infty$$

$$y \rightarrow x - 1 \text{ from above}$$

$$\text{let } x = -1000$$

$$g(x) = -1001$$

$$f(-1000) = -1001.002$$

$$x \rightarrow -\infty$$

$$y \rightarrow x - 1 \text{ from below}$$