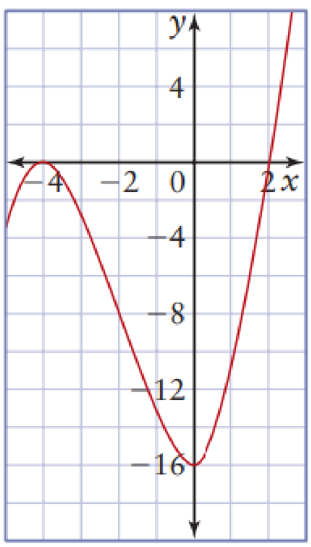
MCV4U Homework

Concavity and Points of Inflection

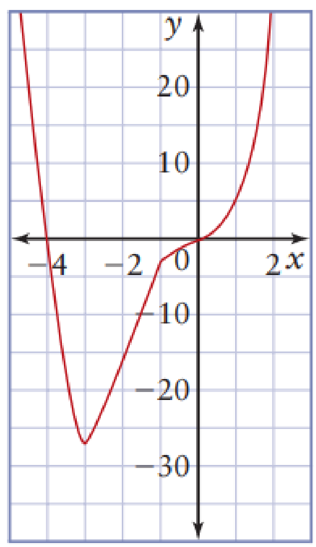
Section A: Short Questions

1. For each graph, identify the intervals over which the graph is concave up and the intervals over which it is concave down.

(a).

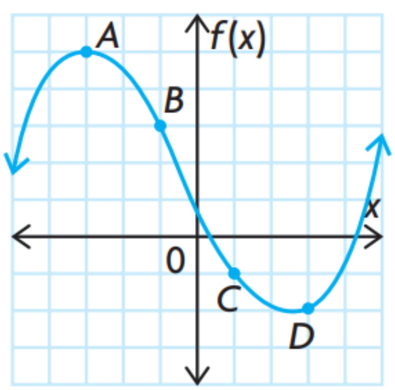


(b).

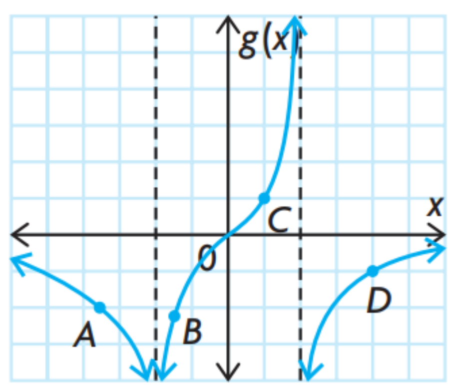


2. For each function, state whether the value of the second derivative is positive or negative at each of points A, B, C and D.

(a).

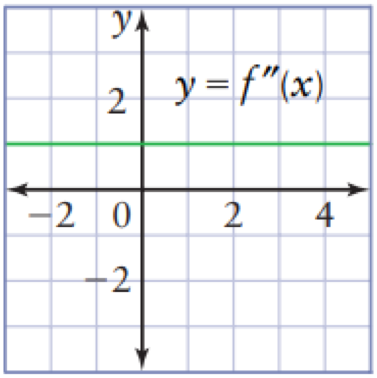


(b).

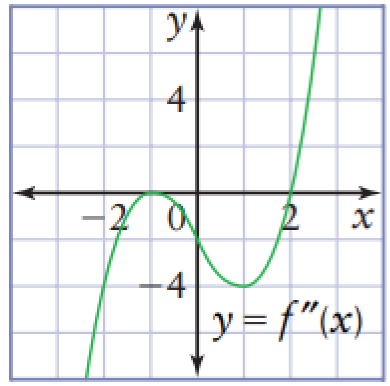


3. Given each graph of $f''(x)$, state the intervals of concavity for the function $f(x)$. Also indicate where any points of inflection will occur.

(a).



(b).



Section B: Problem-solving Questions

4. Determine the value of the second derivative at the value indicated. State whether the curve lies above or below the tangent at this point.

a. $f(x) = 2x^3 - 10x + 3$ at $x = 2$

b. $g(x) = x^2 - \frac{1}{x}$ at $x = -1$

5. Determine the critical points for each function, and use the second derivative test to decide if the point is a local maximum, a local minimum, or neither.

a. $h(x) = x^3 - 6x^2 - 15x + 10$

b. $y = (x - 3)^3 + 8$

6. Find constants $a, b,$ and c such that the function $f(x) = ax^3 + bx^2 + c$ will have a local extremum at $(2, 11)$ and a point of inflection at $(1, 5)$.

7. Describe how you would use the second derivative to determine a local minimum or maximum.

Answer Key

1. a) Intervals of concave up: $x > -2$

   Internals of concave down: $x < -2$

   b) Intervals of concave up: $x < -2$ and $x > 0$

   Intervals of concave down: $-2 < x < 0$

2. a) A: negative, B: negative, C: positive, D: positive

   b) A: negative, B: negative, C: positive, D: negative

3. a) The function is concave up for all real numbers. There are no points of inflection.

   b) Intervals of concave up: $x > 2$

   Intervals of concave down: $x < -1$ and $-1 < x < 2$

   Point of inflection: $x = 2$

4. a) 24; above

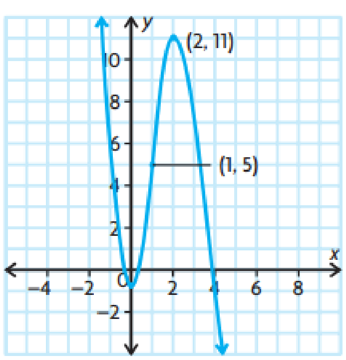
   b) 4; above

5. a) Local minimum = $(5, -105)$

   Local maximum = $(-1, 20)$

   b) $(3, 8)$ is neither a local maximum or minimum

6. $a = -3,\ b = 9,\ c = -1$



7. Use the x values of the critical points that you get from equating first derivative to 0 and plug it into second derivative and if it result in a value lesser than 0, then it's concave down(maximum) and if it result in value greater than 0, then it's concave up(minimum).