Grade 12 Calculus and Vectors MCV4U
Chapter 4 Quiz 2

Section A: Multiple Choice Questions

1. What is a critical point of a function $f(x)$? (1K)

   A). A point where $f(x) = 0$

   B). A point where $f'(x) = 0$ or $f'(x)$ does not exist

   C). A point where $f''(x) = 0$

   D). A point where the graph has a vertical asymptote

2. The graph of a function is increasing, then decreasing. What does this suggest about the point where the change occurs? (1K)

   A). Point of inflection

   B). Local minimum

   C). Asymptote

   D). Local maximum

3. Which of the following is **not** a correct method for finding horizontal asymptotes of a rational function? (1K)

   A). Compare the highest degrees in numerator and denominator

   B). Set numerator equal to denominator and solve

   C). Divide leading coefficients if degrees are equal

   D). If degree of numerator < denominator, H.A. is $y = 0$

4. The First Derivative Test helps determine: (1K)

   A). The concavity of a function

   B). The existence of asymptotes

   C). The intervals of increase/decrease and classify local extrema

   D). Whether a function is odd or even

Section B: Problem-solving Questions

5. For the following function $g(t) = \frac{3t^2+4}{t^2-1}$, determine the equations of any horizontal asymptotes. Then state whether the curve approaches the asymptote from above or below. (3A)

6. Describe the behavior of a function in the vicinity of an oblique asymptote. Draw a diagram for your explanation. (2C)

7. Determine the critical points of the function $f(x) = x^4 - 8x^3 + 18x^2$ and classify them into maximum, minimum or horizontal tangent without using the second derivative test. (4A)

Solutions

1. B). A point where $f'(x) = 0$ or $f'(x)$ does not exist

2. D). Local maximum

3. B). Set numerator equal to denominator and solve

4. C). The intervals of increase/decrease and classify local extrema

5. $g(t) = \frac{3t^2+4}{t^2-1}$

   $\lim\limits_{t \to \infty} \frac{3t^2+4}{t^2-1} = \frac{3t^2}{t^2} = 3$
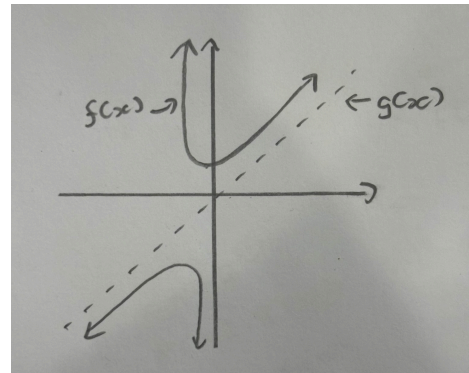
   Horizontal asymptote: $g(t) = 3$

   If $g(1000) \approx 3.000007$ If $g(-1000) \approx 3.000015$

       ✅ From above HA   ✅ From below HA

6. ✅ $\lim\limits_{x \to \pm\infty} f(x) = g(x)$

   Either from above or below

7. $f(x) = x^4 - 8x^3 + 18x^2$

   $f'(x) = 4x^3 - 24x^2 + 36x$

   When $f'(x) = 0$, $4x^3 - 24x^2 + 36x = 0$

   $4x(x^2 - 6x + 9) = 0$

   $4x(x - 3)^2 = 0$

   Critical points: $x = 0$ and $x = 3$

   Verify if the critical points are defined by substituting in $f(x)$.

   $f(0) = 0$ & $f(3) = 27$

-1	$f'(x) = 0$	$+1$	Nature
$f'(-1) = -64$	0	$f'(1) = 16$	$-\ 0\ +$
$f'(2) = 8$	3	$f'(4) = 16$	$+\ 0\ +$

✅ $(0, 0)$ is minimum point and $(3, 27)$ is a horizontal tangent.