Grade 12 Calculus and Vectors MCV4U

Chapter 4 Quiz 3

Section A: Multiple Choice Questions

1. What does the second derivative tell you about a function? (1K)

    A). Whether the function is increasing or decreasing

    B). Whether the graph is above or below the x-axis

    C). The concavity of the graph

    D). The slope of the tangent line

2. A point of inflection occurs when: (1K)

    A). $f'(x) = 0$

    B). $f(x) = 0$

    C). The function has a local maximum

    D). The sign of $f''(x)$ changes

3. Which of the following must be true at a point of inflection? (1K)

    A). $f''(x) = 0$ and $f'(x) = 0$

    B). $f'(x) = 0$ and $f''(x) > 0$

    C). $f''(x)$ changes sign

    D). $f(x) = 0$

4. A function has a local maximum at $x = 3$. Which of the following must be true? (1K)

    A). $f'(3) = 0$ and $f''(3) > 0$

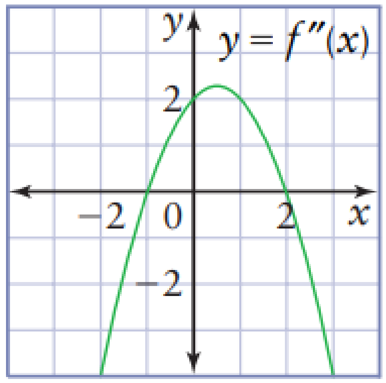
    B). $f'(3) = 0$ and $f''(3) < 0$

    C). $f'(3) > 0$ and $f''(3) < 0$

    D). $f''(3) = 0$ and $f'(3) < 0$

Section B: Problem-solving Questions

5. Given the graph of $f''(x)$, state the intervals of concavity for the function $f(x)$. Also indicate where any points of inflection will occur. (3T)



6. For the function $f(x) = x^4 - 6x^2 - 5$, find the points of inflection and the intervals of concavity. (4K)

7. Find the critical points of the function $h(x) =- 6x^3 + 18x^2 + 3$ and classify them using second derivative test. (3K)

8. Use the algorithm for curve sketching to sketch the following function $f(x) = x^3 - 3x^2$.(4T)

Solutions

1. C). The concavity of the graph
2. D). The sign of $f''(x)$ changes
3. C). $f''(x)$ changes sign
4. B). $f'(3) = 0$ and $f''(3) < 0$
5. The function $f(x)$ is concave up for $-1 < x < 2$ and concave down for $x < -1$ and $x > 2$. There are points of inflection when $x = -1$ and $x = 2$.
6. $f''(x) = 12x^2 - 12$

   When $f''(x) = 0,\ 12(x^2 - 1) = 0$

   $$x = \pm 1$$

   $f(-1) = -10, f(1) = -10$

   ∴ Points of Inflection = $(-1, -10)$ and $(1, -10)$

	$(-\infty, -1)$	$x = -1$	$(-1, 1)$	$x = 1$	$(1, \infty)$
$f''(x)$	When $x = -2$, then $+$	0	When $x = 0$, then $-$	0	When $x = 2$, then $+$
$f(x)$	Concave Up	Inflection Point	Concave Down	Inflection Point	Concave Up

   ∴ $f(x)$ is concave up over the intervals $(-\infty, -1) \cup (1, \infty)$ and concave down over the intervals $(-1, 1)$.

7. $h'(x) = -18x^2 + 36x$

   When $h'(x) = 0, 18x(2 - x) = 0$

   Critical numbers: $x = 0$ and $x = 2$

   $h(0) = 3,\qquad h(2) = 27$ Critical points defined

   $h''(x) = -36x + 36$

   When $x = 0,\ h''(0) = 36 > 0$ Concave up ∴ minimum

When $x = 2, h''(2) =- 36 < 0$ Concave down ∴ maximum

✅ $(0, 3)$ → minimum point
✅ $(2, 27)$ → maximum point

8. ①

$f(x) = x^3 - 3x^2$

y int, $x = 0$

$y = 0$

$(0,0)$

x int, $y = 0$

$x^3 - 3x^2 = 0$

$x^2(x - 3) = 0$

$x = 0$ (or) $x = 3$

$(0,0)$ $(3,0)$

② $f'(x) = 3x^2 - 6x$

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$, $x = 2$

$(0,0)$ $(2,-4)$

③ $f''(x) = 6x - 6$

at $x = 0$

$f''(0) = -6 < 0$

∩ max

$(0,0)$

at $x = 2$

$f''(2) = 6 > 0$

∪ min

$(2,-4)$

④ Inflection

$f''(x) = 6x - 6$

$f''(x) = 0$

$6x - 6 = 0$

$x = 1$ $(1,-2)$

Interval	$x<1$	$x=1$	$x>1$
$f''(x)$	$x=0$ $f''(0) = -6<0$	0	$x = 2$ $f''(2) = 6>0$
$f(x)$	∩	I.P	∪