

Homework

- 1) Sketch the curve of the function $f(x) = x^2 - 4x + 3$.
- 2) Sketch the curve of the function $f(x) = x^3 - 3x$.
- 3) Sketch the curve of the function $f(x) = x^2 - 4$.
- 4) Sketch the curve of the function $f(x) = x^4 - 4x^2$.
- 5) Sketch the curve of the function $f(x) = x^3 - 9x^2 + 15x + 30$.
- 6) Sketch the curve of the function $f(x) = -3x^2 - 2x^2 + 5x$.

Answer Key

1)

① $f(x) = x^2 - 4x + 3$

y int
 $x=0, y=3$

$(0, 3)$

x int
 $y=0, (x-3)(x-1)=0$
 $x=3$ and $x=1$
 $(3, 0), (1, 0)$

② $f'(x) = 2x - 4$

$2x - 4 = 0$

$2x = 4$

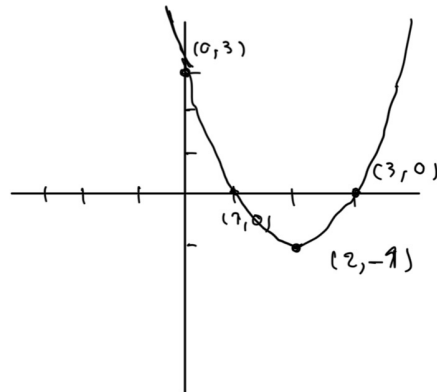
$x = 2$

④ $f''(x) = 0$
 $2 = 0$ No inflection point

③ $f'(x) = 2$

$f''(2) = 2 > 0$

U min $(2, -1)$



2)

① $f(x) = x^3 - 3x$

y int
 $x=0, y=0$
 $(0,0)$

x int
 $y=0, x(x^2-3)=0$
 $x=0$ and $x=\pm\sqrt{3}$
 $(0,0), (1.73,0), (-1.73,0)$

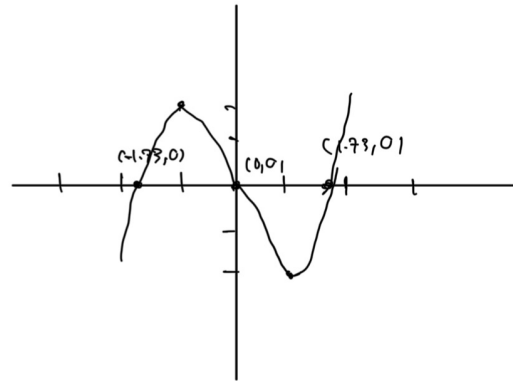
②
 $f'(x) = 3x^2 - 3$
 $3x^2 - 3 = 0$
 $x^2 = 1$
 $x = \pm 1$

③
 $f''(x) = 6x$
 At $x=1$
 $f''(1) = 6 > 0$

∇ min $(1, -2)$

At $x=-1$
 $f''(-1) = -6 < 0$
 $\nearrow$ max $(-1, 2)$

④ Inflection point
 $f''(x) = 0$
 $6x = 0$
 $x = 0$ $(0,0)$



3)

① $f(x) = x^2 - 4$

y int

$x=0, y=-4$

$(0, -4)$

x int

$y=0, (x-2)(x+2)=0$

$x=2$ and $x=-2$

$(2, 0) (-2, 0)$

②

$f'(x) = 2x$

$2x = 0$

$x = 0$ (Potential crib point)

③ $f''(x) = 2$

$f''(0) = 2 > 0$

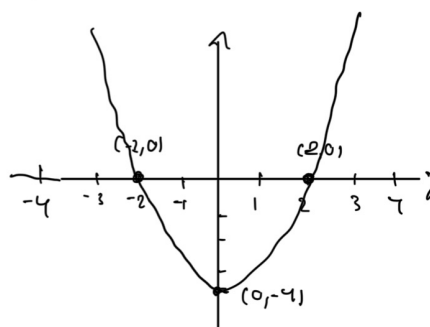
$\cup$ min

$(0, -4)$ is min

④ Inflection point

$f''(x) = 0$

$2 = 0$ no inflection point



4)

$$f(x) = x^4 - 4x^2$$

①

y fnt

sc fnt, y=0

$$x=0, y=0$$

$$x^2(x^2-4)=0$$

$$(0,0)$$

$$x=0 \text{ or } x=\pm 2$$

$$(0,0), (2,0), (-2,0)$$

$$② f'(x) = 4x^3 - 8x$$

$$4x^3 - 8x = 0$$

$$4x(x^2-2)=0$$

$$x=0, x=\pm\sqrt{2}$$

$$(0,0), (\sqrt{2}, -4), (-\sqrt{2}, -4)$$

$$③ f''(x) = 12x^2 - 8$$

$$\text{At } x=0$$

$$f''(0) = -8 < 0$$

$\overline{\wedge}$ Max

$$(0,0)$$

$$\text{At } x=\sqrt{2}$$

$$f''(\sqrt{2}) = 16 > 0$$

$\vee$ min

$$(\sqrt{2}, -4)$$

$$\text{At } x=-\sqrt{2}$$

$$f''(-\sqrt{2}) = 16 > 0$$

$\vee$ min

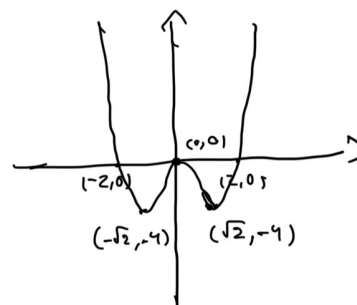
$$(-\sqrt{2}, -4)$$

$$④ 12x^2 - 8 = 0$$

$$x = \pm \frac{\sqrt{6}}{3}$$

Interval	$x < -\frac{\sqrt{6}}{3}$	$x = -\frac{\sqrt{6}}{3}$	$x > -\frac{\sqrt{6}}{3}$
$f''(x)$	> 0	0	< 0
$f(x)$	$\vee$	Inflection	$\wedge$

Interval	$x < \frac{\sqrt{6}}{3}$	$x = \frac{\sqrt{6}}{3}$	$x > \frac{\sqrt{6}}{3}$
$f''(x)$	< 0	0	> 0
$f(x)$	$\wedge$	IP	$\vee$



5)

$$f(x) = x^3 - 9x^2 + 15x + 30$$

$$y \text{ int} \\ (0, 30)$$

$$f'(x) = 3x^2 - 18x + 15$$

$$3x^2 - 18x + 15 = 0$$

$$x^2 - 6x + 5 = 0$$

$$(x-1)(x-5) = 0$$

$$x=1 \text{ or } x=5$$

$$(1, 37) \quad (5, 5)$$

$$f''(x) = 6x - 18$$

$$\text{at } x=1$$

$$f''(1) = -12 < 0$$

$$\cap \text{ max } (1, 37)$$

$$\text{at } x=5$$

$$f''(5) = 12 > 0$$

$$\cup \text{ min } (5, 5)$$

Inflection

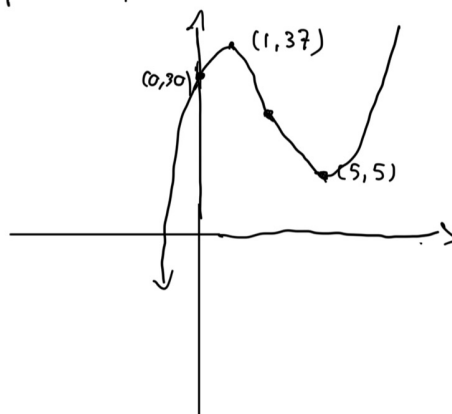
$$f''(x) = 0$$

$$6x - 18 = 0$$

$$x = 3$$

$$(3, 21)$$

Interval	$x < 3$	$x = 3$	$x > 3$
$f''(x)$	< 0	0	> 0
$f(x)$	$\cap$	Inflection	$\cup$



6)

① $f(x) = -3x^3 - 2x^2 + 5x$

y int x int
 $x=0, y=0$ $y=0, -3x^3 - 2x^2 + 5x = 0$
 $(0,0)$ $-x(3x^2 + 2x - 5) = 0$
 $-x(3x+5)(x-1) = 0$
 $x=0, x=-\frac{5}{3}, x=1$
 $(0,0) (-\frac{5}{3}, 0) (1, 0)$

④ $f'(x) = 0$
 $-18x - 4 = 0$
 $x = -\frac{2}{9}$

Interval	$x < -\frac{2}{9}$	$x = 0$	$x > -\frac{2}{9}$
$f'(x)$	> 0	0	< 0
$f(x)$	$\cup$	Inflection	$\cap$

② $f'(x) = -9x^2 - 4x + 5$

$-9x^2 - 4x + 5 = 0$
 $-(9x^2 + 4x - 5) = 0$
 $-(9x - 5)(x + 1) = 0$
 $x = \frac{5}{9}$ (or) $x = -1$

③ $f'(x) = -18x - 4$ $x = -1$
 $x = \frac{5}{9}$ $f'(-1) = 14 > 0$
 $f'(\frac{5}{9}) = -14 < 0$ $\nabla \min$
 $\nearrow \max$ $(-1, -4)$
 $(\frac{5}{9}, 1.65)$

